JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 4, July–August 1994

Aircraft Model for the AIAA Controls Design Challenge

Randal W. Brumbaugh*
PRC Inc., Edwards, California, 93523

This paper describes a generic, state-of-the-art, high-performance aircraft model, including detailed, full-envelope, nonlinear aerodynamics, and full-envelope thrust and first-order engine response data. Although this model was developed primarily for the AIAA Controls Design Challenge, the availability of such a model provides a common focus for research in aeronautical control theory and methodology. Figures showing vehicle geometry, surfaces, and sign conventions are included.

Nomenclature

A = axial force, lb
a = speed of sound in air, ft/s
a_n = normal acceleration, g
a_{nx} = x-body axis accelerometer output, accelerometer at center of gravity, g
a_{ny} = y-body axis accelerometer output, accelerometer at center of gravity, g
a_{nz} = z-body axis accelerometer output, accelerometer at center of gravity, g
a_x = acceleration along the x-body axis, g
a_y = acceleration along the y-body axis, g
a_z = acceleration along the z-body axis, g
b = wingspan, ft
C = force or moment coefficient
C_D = coefficient of drag
C_L = coefficient of lift
C_ℓ = coefficient of rolling moment
C_m = coefficient of pitching moment
C_n = coefficient of yawing moment
C_y = coefficient of sideforce
$\bar{c}$ = mean aerodynamic chord, ft
D = drag force, lb
E_s = specific energy, ft
F_{pa} = Flightpath acceleration
G = actuator transfer function
g = acceleration due to gravity, ft/s^2
h = altitude, ft
$\ddot{h}$ = vertical acceleration, ft/s^2
I = aircraft inertia tensor, slug-ft^2
I_e = rotational inertia of the engine, slug-ft^2
I_x = x-body axis moment of inertia, slug-ft^2
I_{xy} = x-y body axis product of inertia, slug-ft^2
I_{xz} = x-z body axis product of inertia, slug-ft^2
I_y = y-body axis moment of inertia, slug-ft^2
I_{yz} = y-z body axis product of inertia, slug-ft^2
I_z = z-body axis moment of inertia, slug-ft^2
K = aerodynamic constant
L = total body axis aerodynamic rolling moment, ft-lb; or total aerodynamic lift, lb
ℓ = generalized length, ft
M = Mach number; or total body axis aerodynamic pitching moment, ft-lb
m = aircraft total mass, slug
m_e = mass of engine
N = normal force, lb; or total body axis aerodynamic yawing moment, ft-lb
n = load factor
P_s = specific power, ft/s
p = roll rate, rad/s; or pressure, lb/ft^2
p_a = ambient pressure, lb/ft^2
p_t = total pressure, lb/ft^2
q = pitch rate, rad/s
$\bar{q}$ = dynamic pressure, lb/ft^2
q_c = impact pressure, lb/ft^2
Re = Reynolds number
Re' = Reynolds number per unit length, ft^{-1}
r = yaw rate, rad/s
S = wing planform area, ft^2
s = complex frequency
T = ambient temperature, K; or total angular momentum slug-ft^2/s^2; or thrust, lb
T_t = total temperature, K
u = velocity in x-axis direction, ft/s
V = total velocity, ft/s
V_c = calibrated airspeed, kt
V_e = equivalent airspeed, kt
v = velocity in y-axis direction, ft/s
W = vehicle weight, lb
w = velocity in z-axis direction, ft/s
X = total force along the x-body axis, lb
X_T = thrust along the x-body axis, lb
Y = sideforce, lb
Y_T = thrust along the y-body axis, lb
Z = total force along the z-body axis, lb
Z_T = thrust along the z-body axis, lb
α = angle of attack, rad
β = angle of sideslip, rad
γ = flightpath angle, rad
Δr = displacement of aerodynamic reference point from center of gravity
Δx = displacement from center of gravity along x-body axis, ft
Δy = displacement from center of gravity along y-body axis, ft
Δz = displacement from center of gravity along z-body axis, ft
δ_A = differential aileron command
δ_D = differential stabilator command
δ_H = symmetric stabilator command
δ_R = directional command
$\delta_j k$ = Kronecker delta
θ = pitch angle, rad
μ = coefficient of viscosity
ρ = density of air, slug/ft
ΣL = total body axis rolling moment, ft-lb
ΣM = total body axis pitching moment, ft-lb
ΣN = total body axis yawing moment, ft-lb

Presented as Paper 91-2631 at the AIAA Guidance, Navigation and Control Conference, New Orleans, LA, Aug. 12–15, 1991; received April 25, 1992; revision received Oct. 14, 1993; accepted for publication Oct. 15, 1993.

*Electrical Engineer. Member AIAA.

ϕ = roll angle, rad
ϕ_L = tilt angle of acceleration normal to the flightpath from the vertical plane, rad
ψ = heading angle, rad
ω = total rotational velocity of the vehicle

Superscript

$\cdot$ = derivative with respect to time

Subscripts

A = aileron
ar = aerodynamic reference point
D = differential stabilator
D = total drag
E = engine
H = symmetric stabilator
h = altitude
$,i$ = measurement not at aerodynamic reference
L = total lift
ℓ = rolling moment
M = Mach number
m = pitching moment
n = yawing moment
o = offset from center of gravity
p = roll rate
q = pitch rate
R = rudder
r = yaw rate
s = stability axis
x = along the x-body axis
Y = sideforce
y = along the y-body axis
z = along the z-body axis
0 = standard day, sea-level conditions

Introduction

THIS paper describes the structure and implementation of a high-performance aircraft model. The model was developed for the AIAA Controls Design Challenge, but it is intended to be useful for a variety of controls and guidance applications. Model definition and implementation are covered in separate sections.

The AIAA Controls Design Challenge provides an opportunity for participants to apply control system design methodologies to a realistic, nonlinear aircraft model. Any design that performs the control task is acceptable, but innovative or unusual approaches have been encouraged. The challenge is a 2-year competition. Control designs are judged according to their ability to control the model during a level acceleration and 3-g turn maneuver at four specified flight conditions.

The model integrates several components. Existing pieces were used whenever possible and modified to facilitate integration. The result is that most of the model implementation is based directly on proved and reliable components although the resulting model is not completely representative of any particular aircraft. Because of the mixed history of the model, some aspects may seem to resemble actual aircraft. The user is warned against making any assumptions based on these semblances.

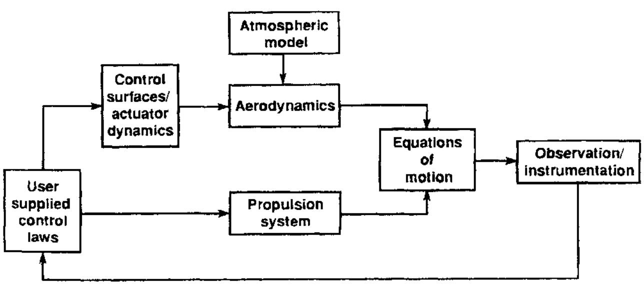

Fig. 1 Modular structure of the model.

Model Characteristics

The model is a collection of modules, each performing a specific function. The primary modules are the aircraft actuator and surface command inputs, aircraft mass and geometry modeling, the equations of motion, the atmospheric model, the aerodynamics, the propulsion system, and the observation variable modeling. Each major module is described in the following sections. Figure 1 shows how the modules would be connected together with user synthesized control laws to form a complete system model.

Aircraft Description

The aircraft modeled is a high-performance, supersonic vehicle representative of current fighters. It is powered by two afterburning turbofan engines, each capable of producing approximately 32,000 lb of thrust. A three-view of the aircraft is shown in Fig. 2, including control surfaces and locations.

The aircraft primary flight-control surfaces consist of horizontal stabilators capable of symmetric or differential movement, conventional ailerons, and a single vertical rudder. The individual surface position limits, rate limits, and sign conventions for positive deflection are detailed in Table 1. The equations in Table 2 define the individual surface deflections in terms of command inputs. There are a total of five actuators; two aileron, two stabilator, and one rudder. The model includes identical actuators for all surfaces. These actuators are rate limited at 24 deg/s and have a first-order response modeled by $G(s) = 20/(s + 20)$.

A block diagram of the actuator model is shown in Fig. 3. The command inputs to the aileron and stabilator surfaces are

Table 1 Command input limits and sign conventions

Command name	Symbol	Limits, deg	Positive sign convention
Aileron	δ_A	± 20	Left trailing edge down
Symmetric stabilator	δ_H	$+15/-25$	Trailing edge down
Differential stabilator	δ_D	± 20	Left trailing edge down
Rudder	δ_R	± 30	Trailing edge left

Table 2 Surface deflection definition equations

Surface	Deflection definition	Rate limit, deg/s
$\delta_{A_{left}}$	$\delta_A \div 2$	24
$\delta_{A_{right}}$	$-\delta_A \div 2$	24
$\delta_{H_{left}}$	$(2\delta_H - \delta_D) \div 2$	24
$\delta_{H_{right}}$	$(2\delta_H + \delta_D) \div 2$	24
δ_R	δ_R	24

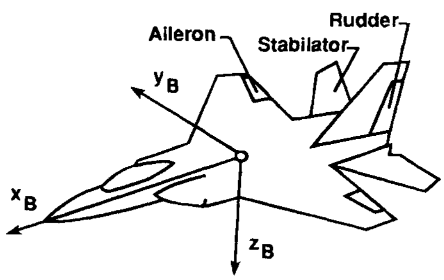

Fig. 2 Three view of aircraft and control surfaces.

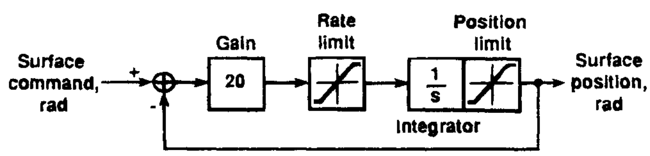

Fig. 3 Block diagram of actuator model.

differential and symmetric commands, which are separated into inputs to each of the surface actuator models. The resulting surface positions are then recombined to obtain the command response. This is shown for the stabilators in Fig. 4. Because of the nonlinearities in the stabilator command path, the commands will interact in ways not easily predicted. This interaction is shown in Figs. 5 and 6.

The operational envelope for this vehicle, for trimmed, straight-and-level 1-g flight is shown in Fig. 7 for the specified, weight of 45,000 lb. The envelope includes a maximum Mach number of 2.3 and an altitude limit in the 50,000- to 60,000-ft range. Mass and geometry parameters are given in Table 3.

Aerodynamic Model

The aerodynamics are modeled for the full vehicle envelope using multidimensional tables and linear interpolation to form nonlinear function generators. In general, these aerodynamic quantities are functions of M and some combination of angle of attack, angle of sideslip, and symmetric stabilator deflection.

The equations defining the aerodynamic model provide nondimensional force and moment coefficients. The longitudinal parameters are in the stability axis system; the lateral–directional parameters are given with respect to the body axis system.

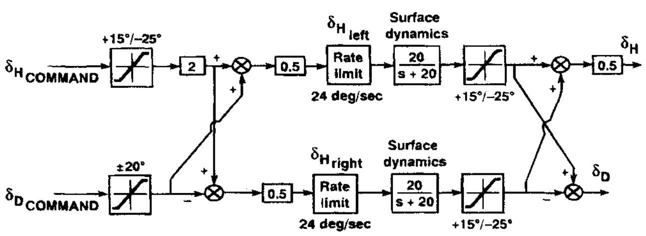

Fig. 4 Stabilator command path.

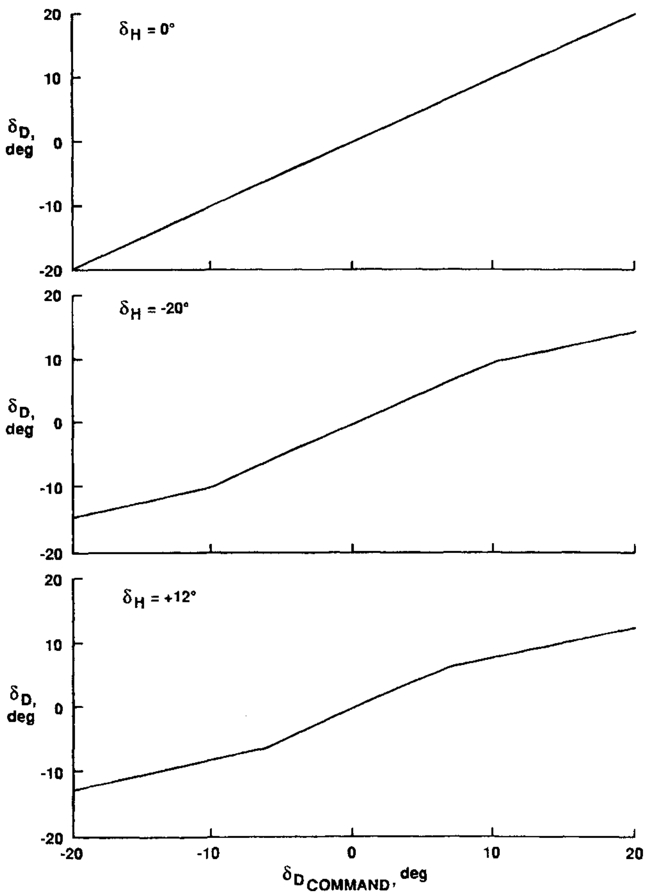

Fig. 5 Effect of δ_H on δ_D response.

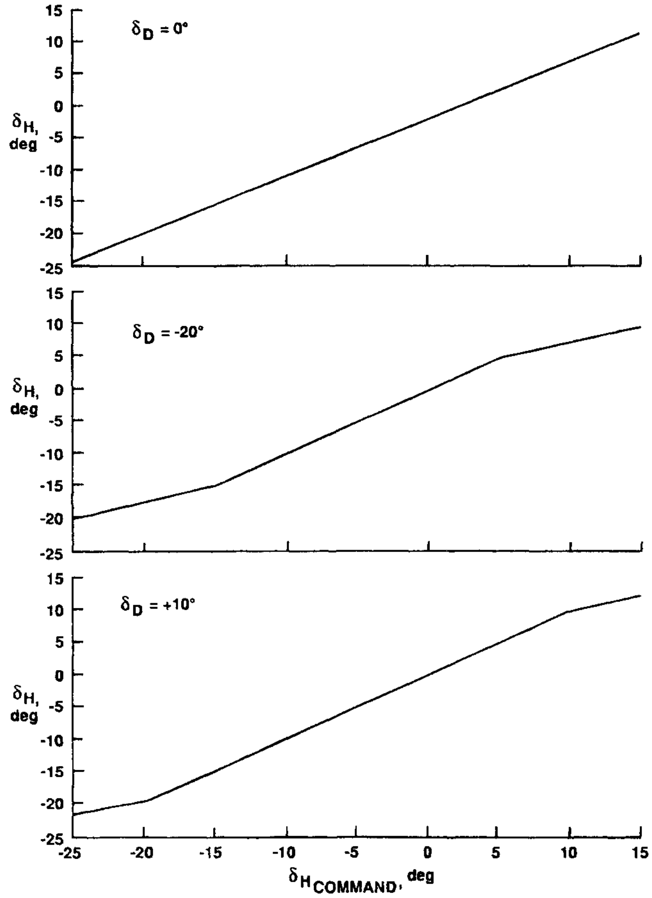

Fig. 6 Effect of δ_D on δ_H response.

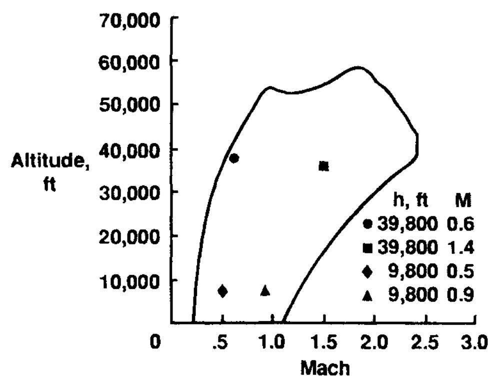

Fig. 7 Vehicle operational envelope at 45,000 lb.

Table 3 Mass and geometry characteristics

Parameter	Symbol	Value
Wing area	S	608.0 ft²
Wing span	b	42.8 ft
Mean aerodynamic chord	$\bar{c}$	15.95 ft
Vehicle weight	W	45,000.0 lb
Moments of inertia		
Roll	I_x	28,700.0 slug/ft²
Pitch	I_y	165,100.0 slug/ft²
Yaw	I_z	187,900.0 slug/ft²
Products	I_{xz}	−520.0 slug/ft²
	I_{xy}	0.0 slug/ft²
	I_{yz}	0.0 slug/ft²

The equations used for this model are as follows:

$$C_{L_{\mathrm{STAB}}} = C_{L_{\mathrm{BASIC}}} + \Delta C_{L_{n_z}} n_z$$

$$C_{m_{\mathrm{STAB}}} = C_{m_{\mathrm{BASIC}}} + \Delta C_{m_{n_z}} n_z + \bar{c}/(2V) \times (C_{m_q} q + C_{m_{\dot\alpha}} \dot\alpha + C_{L_{\mathrm{BASIC}}} \Delta N_0)$$

$$C_{y_{\mathrm{BODY}}} = C_{y_{\mathrm{BASIC}}} + C_{y_{\delta_A}} \delta_A + C_{y_{\delta_D}} \delta_D - \Delta C_{y_{\delta_R}} K_{\delta_{R_y}}$$

$$C_{\ell_{\mathrm{BODY}}} = C_{\ell_{\mathrm{BASIC}}} + C_{\ell_{\delta_A}} \delta_A + C_{\ell_{\delta_D}} \delta_D - \Delta C_{\ell_{\delta_R}} K_{\delta_{R\ell}} + b/(2V)(C_{\ell_p} p + C_{\ell_r} r)$$

$$C_{n_{\mathrm{BODY}}} = C_{n_{\mathrm{BASIC}}} + C_{n_{\delta_A}} \delta_A + C_{n_{\delta_D}} \delta_D - \Delta C_{n_{\delta_R}} K_{\delta_{Rn}} + b/(2V)(C_{n_p} p + C_{n_r} r)$$

The terms in the equations containing C, ΔC, ΔN, or K are outputs from the function generation routines, and are either calculated by linear interpolation of tabular data or by direct calculation. The source of the functional coefficients is shown in Table 4.

Propulsion System Model

The propulsion system model consists of two distinct engine models. The engines are similar, but not identical; the thrust produced for identical throttle settings is not symmetrical. Each engine thrust vector is aligned with the body axis, and acts at a point located 10 ft behind the vehicle center of gravity and 4 ft laterally from the centerline. The thrust produced by each engine is a function of altitude, M, and throttle setting. Each engine is modeled as a nonlinear system having two sections: a core engine and an afterburner (augmentor) section with associated sequencing logic.

Throttle position inputs to the engine model are in degrees, with a minimum position of 20 deg and a maximum of 127 deg. The core section responds to throttle inputs up to 83 deg. The afterburner section begins to respond at a throttle position of 91 deg. The core model has first-order dynamics and rate limiting to model spool-up effects. A block diagram of the core model dynamics is shown in Fig 8. The afterburner has a rate limiter and sequencing logic to model the fuel pump and pressure regulator effects. A block diagram of the afterburner model dynamics is shown in Fig. 9.

Table 4 Source of aerodynamic coefficients

Coefficient	Source	Independent variables
$C_{L_{\mathrm{BASIC}}}$	Table	M, α, δ_H
$\Delta C_{L_{n_z}}$	Table	M
$C_{m_{\mathrm{BASIC}}}$	Table	M, α, δ_H
$\Delta C_{m_{n_z}}$	Table	M
C_{m_q}	Table	M, α
$C_{m_{\dot\alpha}}$	Table	M, α
ΔN_0	Table	M
$C_D(\alpha < 32)$	Table	$C_{L_{\mathrm{BASIC}}}$, M
$(32 < \alpha < 40)$	Table	$C_{L_{\mathrm{BASIC}}}$, M, α
$(\alpha > 40)$	Calc	$C_{L_{\mathrm{BASIC}}}$, α
$\Delta C_{D_{\mathrm{alt}}}$	Calc	h
$\Delta C_{D_{\mathrm{nozzle}}}$	Table	M, throttle
$C_{\ell_{\mathrm{BASIC}}}$	Table	M, α, β
$\Delta C_{\ell_{\delta_R}}$	Table	M, α, δ_R
$K_{\delta_{R\ell}}$	Table	M
C_{ℓ_p}	Table	M, α
C_{ℓ_r}	Table	M, α
$C_{n_{\mathrm{BASIC}}}$	Table	M, α, β
$C_{n_{\delta_A}}$	Table	M, α
$C_{n_{\delta_D}}$	Table	M, α
$\Delta C_{n_{\delta_R}}$	Table	M, α, β
$K_{\delta_{Rn}}$	Table	M, α
C_{n_p}	Table	M, α
C_{n_r}	Table	M, α
$C_{y_{\mathrm{BASIC}}}$	Table	M, α, β
$C_{y_{\delta_A}}$	Table	M, α
$C_{y_{\delta_D}}$	Table	M, α
$\Delta C_{y_{\delta_R}}$	Table	M, α, δ_R
$K_{\delta_{Ry}}$	Table	M

Observation Model

The observation variables provided by this model represent a broad class of parameters useful for vehicle analysis and control design problems. These variables include the state, time derivatives of state, and control variables. Airdata parameters, accelerations, flightpath terms, and other miscellaneous parameters are also included. The equations used to calculate those parameters are derived from a number of sources.[1-5,7] Implicit in many of these observation equations is an atmospheric model. The atmospheric model is derived from the U.S. Standard Atmosphere.[6]

Three body axis angular rates and three translational accelerations are available as observation variables. These include the x-body axis rate, the y-body axis rate, and the z-body axis rate. The time derivatives of these quantities, $\dot u$, $\dot v$, and $\dot w$, are also included. The equations defining these quantities are

$$u = V \cos\alpha \cos\beta$$

$$v = V \sin\beta$$

$$w = V \sin\alpha \cos\beta$$

$$\dot u = (X_T - gm \sin\theta - D\cos\alpha + L\sin\alpha)/m + rv - qw$$

$$\dot v = (Y_T + gm\cos\theta\sin\phi + Y)/m + pw - ru$$

$$\dot w = (Z_T + gm\cos\theta\cos\phi - D\sin\alpha - L\cos\alpha)/m + qu - pv$$

The vehicle body axis accelerations constitute the set of observation variables that, except for state variables themselves, are most commonly used in aircraft control analysis and design problems. These accelerations are measured in g units and are derived directly from the body axis forces defined in the previous section for translational acceleration. The equations used for the body axis acceleration are

$$a_x = (X_T - D\cos\alpha + L\sin\alpha - gm\sin\theta)/(g_0 m)$$

$$a_y = (Y_T + Y + gm\cos\theta\sin\phi)/(g_0 m)$$

$$a_z = (Z_T - D\sin\alpha - L\cos\alpha + gm\cos\theta\cos\phi)/(g_0 m)$$

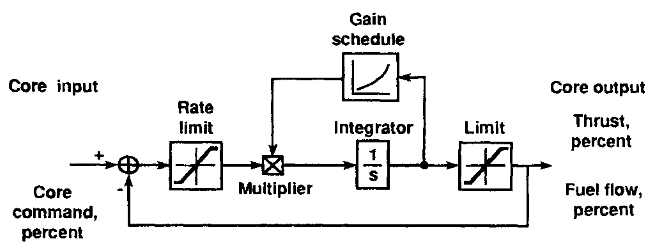

Fig. 8 First order engine core dynamics.

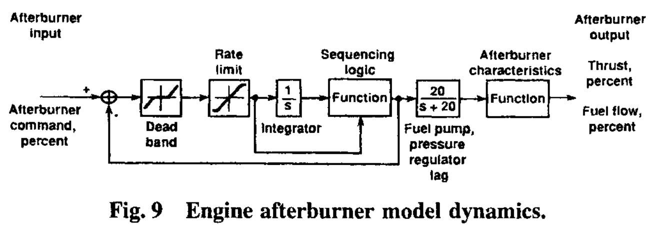

Fig. 9 Engine afterburner model dynamics.

The equations for the outputs of the body axis accelerometers that are at vehicle center of gravity are

$$a_{nx} = (X_T - D\cos\alpha + L\sin\alpha)/(g_0 m)$$

$$a_{ny} = (Y_T + Y)/(g_0 m)$$

$$a_{nz} = (Z_T - D\sin\alpha - L\cos\alpha)/(g_0 m)$$

$$a_n = -a_{nz}$$

For orthogonal accelerometers aligned with the vehicle body axes but are not at the vehicle center of gravity, the following equations apply

$$a_{nx,\,i} = a_{nx} - [(q^2 + r^2)x_x - (pq - \dot{r})y_x - (pr + \dot{q})z_x]/g_0$$

$$a_{ny,\,i} = a_{ny} + [(pq + \dot{r})x_y - (p^2 + r^2)y_y + (qr - \dot{p})z_y]/g_0$$

$$a_{nz,\,i} = a_{nz} + [(pr - \dot{q})x_z + (qr + \dot{p})y_z - (q^2 + p^2)z_z]/g_0$$

$$a_{n,\,i} = -a_{nz,\,i}$$

where the symbols x, y, and z refer to the x-, y-, and z-body axis locations of the sensors relative to the vehicle center of gravity. Included in the set of acceleration equations is load factor $n = L/W$. Included in the observation variables are the flightpath-related parameters, including flightpath angle, flightpath acceleration, verticle acceleration rate ($\dot{\gamma}$), and scaled altitude rate ($\dot{h}/57.3$). The equations used to determine these quantities are

$$\gamma = \sin^{-1}(\dot{h}/V)$$

$$fpa = \dot{V}/g_0$$

$$\ddot{h} = a_x\sin\theta - a_y\sin\phi\cos\theta - a_z\cos\phi\cos\theta$$

$$\dot{\gamma} = (V\ddot{h} - \dot{h}\dot{V})/(V\sqrt{V^2 - \dot{h}^2})$$

Two energy-related terms are included in the observation variables; specific energy and specific power, defined as

$$E_s = h + V^2/(2g)$$

$$P_s = \dot{h} + V\dot{V}/g$$

The set of observation variables available also includes four force parameters: total aerodynamic lift, total aerodynamic drag, total aerodynamic normal force, and total aerodynamic axial force. These quantities are defined as

$$L = \bar{q}SC_{L_{\mathrm{STAB}}}$$

$$D = \bar{q}SC_{D_{\mathrm{STAB}}}$$

$$N = L\cos\alpha + D\sin\alpha$$

$$A = -L\sin\alpha + D\cos\alpha$$

where $C_{D_{\mathrm{STAB}}}$ and $C_{L_{\mathrm{STAB}}}$ are coefficients of drag and lift, respectively.

The airdata parameters having the greatest application to aircraft dynamics and control problems are the sensed parameters and the reference and scaling parameters. The sensed parameters are impact pressure, ambient or freestream pressure, total pressure ambient or freestream temperature, and total temperature. The selected reference and scaling parameters are Mach number, dynamic pressure, speed of sound, Reynolds number, Reynolds number per unit length, and the Mach meter calibration ratio (q_c/p_a). These quantities are defined as

$$a = \sqrt{1.4p_0T/(\rho_0T_0)}$$

$$M = V/a$$

$$Re = \rho V\ell/\mu$$

$$Re' = \rho V/\mu$$

$$\bar{q} = \rho V^2/2$$

$$q_c = \begin{cases}[(1.0 + 0.2M^2)^{3.5} - 1.0]p_a & (M \le 1.0)\\ \{1.2M^2[(5.76M^2)/(5.6M^2 - 0.8)]^{2.5} - 1.0\}p_a & (M > 1.0)\end{cases}$$

$$p_t = p_a + q_c$$

$$T_t = T(1.0 + 0.2M^2)$$

where T is ambient of freestream temperature. Freestream pressure, freestream temperature, and coefficient of viscosity are derived from the U.S. Standard Atmosphere.[6]

Included in the airdata calculations are two velocities computed in knots: equivalent airspeed and calibrated airspeed. The calculations assume that internal units are in the English system. The equation used for equivalent airspeed is

$$V_e = 17.17\sqrt{\bar{q}}$$

derived from the definition of equivalent airspeed,

$$V_e = \sqrt{\frac{2\bar{q}}{\rho_0}}$$

where $\rho_0 = 0.002378$ slug/ft^3 and V_e is converted from fps to kt. Calibrated airspeed is derived from the following definition of impact pressure:

$$q_c = \begin{cases}p_0\{[1.0 + \rho_0/(7.0p_0)V_c^2]^{3.5} - 1\} & (V_c \le a_0)\\ 1.2(V_c/a_0)^2p_0\{5.76/[5.6 - 0.8(a_0/V_c)^2]\}^{2.5} - p_0 & (V_c > a_0)\end{cases}$$

For the case where $V_c \le a_0$, the equation for V_c is

$$V_c = 1479.116\sqrt{(q_c/p_0 + 1.0)^{2/7} - 1.0} \quad (V_c \le a_0)$$

Calibrated airspeed is found using an iterative process for the case where $V_c > a_0$

$$V_c = 582.95174\sqrt{(q_c/p_0 + 1.0)\{1.0 - 1.0/[7.0(V_c/a_0)^2]\}^{2.5}}$$
$$(V_c > a_0)$$

is executed until the change in V_c from one iteration to the next is less than 0.001 kt.

The final set of observation variables provided is a miscellaneous collection of other parameters of interest in analysis and design problems. The first group consists of measurements from sensors not located at the vehicle center of gravity. These represent angle of attack, $\alpha_{,i}$, angle of sideslip, $\beta_{,i}$, altitude, $\dot{h}_{,i}$, and altitude rate, h_i, measurements displaced from the center of gravity by some x-, y-, and z-body axis distances. The equations used to compute these quantities are

$$\alpha_{,i} = \alpha - (qx - py)/V$$

$$\beta_{,i} = \beta + (rx - pz)/V$$

$$h_{,i} = h + x\sin\theta - y\sin\phi\cos\theta - z\cos\phi\cos\theta$$

$$\dot{h}_{,i} = \dot{h} + \dot{\theta}(x\cos\theta + y\sin\phi\sin\theta + z\cos\phi\sin\theta)$$
$$- \dot{\phi}(y\cos\phi\cos\theta - z\sin\phi\cos\theta)$$

The remaining miscellaneous parameters are total angular momentum, T, stability axis roll rate, p_s, stability axis pitch rate, q_s, and stability axis yaw rate, r_s, defined as

$$T = \tfrac{1}{2}(I_x p^2 - 2I_{xy}pq - 2I_{xz}pr + I_y q^2 - 2I_{yz}qr + I_z r^2)$$

$$p_s = p\cos\alpha + r\sin\alpha$$

$$q_s = q$$

$$r_s = -p\sin\alpha + r\cos\alpha$$

Equations of Motion and Atmospheric Model

The nonlinear equations of motion used in this model are general 6-DOF equations representing the flight dynamics of a rigid aircraft flying in a stationary atmosphere over a flat, nonrotating Earth. These equations of motion were derived by Etkin,[3] and the derivation is detailed in Ref. 7. The equations for each variable in the state vector are given in the following.

The following equations for rotational acceleration are used:

$$\begin{aligned}\dot p = [&(\Sigma L)I_1 + (\Sigma M)I_2 + (\Sigma N)I_3 - p^2(I_{xz}I_2 - I_{xy}I_3)\\ &+ pq(I_{xz}I_1 - I_{yz}I_2 - D_z I_3) - pr(I_{xy}I_1 + D_y I_2 - I_{yz}I_3)\\ &+ q^2(I_{yz}I_1 - I_{xy}I_3) - qr(D_x I_1 - I_{xy}I_2 + I_{xz}I_3)\\ &- r^2(I_{yz}I_1 - I_{xz}I_2)]/\det I\end{aligned}$$

$$\begin{aligned}\dot q = [&(\Sigma L)I_2 + (\Sigma M)I_4 + (\Sigma N)I_5 - p^2(I_{xz}I_4 - I_{xy}I_5)\\ &+ pq(I_{xz}I_2 - I_{yz}I_4 - D_z I_5) - pr(I_{xy}I_2 + D_y I_4 - I_{yz}I_5)\\ &+ q^2(I_{yz}I_2 - I_{xy}I_5) - qr(D_x I_2 - I_{xy}I_4 + I_{xz}I_5)\\ &- r^2(I_{yz}I_2 - I_{xz}I_4)]/\det I\end{aligned}$$

$$\begin{aligned}\dot r = [&(\Sigma L)I_3 + (\Sigma M)I_5 + (\Sigma N)I_6 - p^2(I_{xz}I_5 - I_{xy}I_6)\\ &+ pq(I_{xz}I_3 - I_{yz}I_5 - D_z I_6) - pr(I_{xy}I_3 + D_y I_5 - I_{yz}I_6)\\ &+ q^2(I_{yz}I_3 - I_{xy}I_6) - qr(D_x I_3 - I_{xy}I_5 + I_{xz}I_6)\\ &- r^2(I_{yz}I_3 - I_{xz}I_5)]/\det I\end{aligned}$$

Where ΣL, ΣM, and ΣN are the aerodynamic total moments about the x-, y-, and z-body axes, respectively, including power plant induced moments, and

$$\det I = I_x I_y I_z - 2I_{xy}I_{xz}I_{yz} - I_x I_{yz}^2 - I_y I_{xz}^2 - I_z I_{xy}^2$$

$$I_1 = I_y I_z - I_{yz}^2$$

$$I_2 = I_{xy}I_z + I_{yz}I_{xz}$$

$$I_3 = I_{xy}I_{yz} + I_y I_{xz}$$

$$I_4 = I_x I_z - I_{xz}^2$$

$$I_5 = I_x I_{yz} + I_{xy}I_{xz}$$

$$I_6 = I_x I_y - I_{xy}^2$$

$$D_x = I_z - I_y$$

$$D_y = I_x - I_z$$

$$D_z = I_y - I_x$$

The translational acceleration equations used are

$$\begin{aligned}\dot V = [&-D\cos\beta + Y\sin\beta + X_T\cos\alpha\cos\beta + Y_T\sin\beta\\ &+ Z_T\sin\alpha\cos\beta - mg(\sin\theta\cos\alpha\cos\beta\\ &- \cos\theta\sin\phi\sin\beta - \cos\theta\cos\phi\sin\alpha\cos\beta)]/m\end{aligned}$$

$$\begin{aligned}\dot\alpha = [&-L + Z_T\cos\alpha - X_T\sin\alpha + mg(\cos\theta\cos\phi\cos\alpha\\ &+ \sin\theta\sin\alpha)]/Vm\cos\beta + q - \tan\beta(p\cos\alpha + r\sin\alpha)\end{aligned}$$

$$\begin{aligned}\dot\beta = [&D\sin\beta + Y\cos\beta - X_T\cos\alpha\sin\beta + Y_T\cos\beta\\ &- Z_T\sin\alpha\sin\beta + mg(\sin\theta\cos\alpha\sin\beta\\ &+ \cos\theta\sin\phi\cos\beta - \cos\theta\cos\phi\sin\alpha\sin\beta)]/(Vm)\\ &+ p\sin\alpha - r\cos\alpha\end{aligned}$$

where L is total aerodynamic lift.

The equations defining the vehicle attitude rates are

$$\dot\theta = q\cos\phi - r\sin\phi$$

$$\dot\psi = q\sin\phi\sec\theta + r\cos\phi\sec\theta$$

$$\dot\phi = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

The equations defining the Earth-relative velocities are

$$\begin{aligned}\dot h = V(&\cos\beta\cos\alpha\sin\theta - \sin\beta\sin\phi\cos\theta\\ &- \cos\beta\sin\alpha\cos\phi\cos\theta)\end{aligned}$$

$$\begin{aligned}\dot x = V[&\cos\beta\cos\alpha\cos\theta\cos\psi + \sin\beta(\sin\phi\sin\theta\cos\psi\\ &- \cos\phi\sin\psi) + \cos\beta\sin\alpha(\cos\phi\sin\theta\cos\psi\\ &+ \sin\phi\sin\psi)]\end{aligned}$$

$$\begin{aligned}\dot y = V[&\cos\beta\cos\alpha\cos\theta\sin\psi + \sin\beta(\sin\phi\sin\theta\sin\psi\\ &+ \cos\phi\cos\psi) + \cos\beta\sin\alpha(\cos\phi\sin\theta\sin\psi\\ &- \sin\phi\cos\psi)]\end{aligned}$$

The atmospheric data model is based on tables from the U.S. Standard Atmosphere.[6] This model calculates values for speed of sound, acceleration caused by gravity, air density, viscosity, and ambient static pressure and temperature. These values are calculated based on altitude. The tabular data are organized on evenly spaced breakpoints between 0 and 90 km. Linear interpolation is used between table values for altitudes in this range; the extreme values are used for altitudes outside the range.

References

[1] Clancy, L. J., *Aerodynamics*, John Wiley & Sons, New York, 1975.

[2] Dommasch, D. O., Sherby, S. S., and Connolly, T. F., *Airplane Aerodynamics*, 4th ed., Pitman, New York, 1967.

[3] Etkin, B., *Dynamics of Atmospheric Flight*, John Wiley & Sons, New York, 1972.

[4] Gainer, T. G., and Hoffman, S., "Summary of Transformation Equations and Equations of Motion Used in Free-Flight and Wind-Tunnel Data Reduction and Analysis," NASA SP-3070, 1972.

[5] Gracey, W., "Measurement of Aircraft Speed and Altitude," NASA RP-1046, 1980.

[6] U.S. Standard Atmosphere, 1962, U.S. GPO, Washington 25, DC, 1962.

[7] Duke, E. L., Antoniewicz, R. F., and Krambeer, K. D., "Derivation and Definition of a Linear Aircraft Model," NASA RP 1207, Aug. 1988.